The Utilizability Method of Predicting the Long Term Performance of Flat Plate Solar Collectors

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ABSTRACT

Methods of predicting the long-term performance of flat plate solar collectors have been developed. In this paper, both the classical Liu and Jordan's ϕ-curve method and Klein's ϕ chart method are reviewed. Computation results by these two methods for solar collectors used in hot water heating, space cooling and heating applications in Hong Kong are presented. A comparison between the results obtained by the two different approaches shows excellent agreement and the maximum discrepancy observed has been only a few percent.

INTRODUCTION

Flat plate solar collectors are commonly used in space heating and cooling, hot water heating and other types of solar energy systems. In order to predict the long-term performance of such systems for their overall design optimization and economic evaluation, either sophisticated computer simulation techniques[1] or design methods[2, 3] have to be used. The computer simulation approach is in general expensive and time-consuming. Very often, it is desirable to adopt simpler design methods.

The classical approach of predicting the long-term performance of flat plate collectors is to use the ϕ curve or the utilizability method, first developed by Whillier[4] and later generalized by Liu and Jordan.[5, 6] An alternative approach is the ϕ chart method, which was developed by Klein.[7] Both methods can be useful to predict the maximum amount of energy collection in a solar energy system employing flat plate collectors. Recently, it has been observed by Gordon, Govaer and Zarmi[8] that the annual utilizability curves for flat plate collectors based on daily insolation data can be significantly different from the corresponding curves based on hourly data, particularly in the high threshold range. In this paper, the methodology and the mathematical formulations of both the ϕ curve method and the ϕ chart method are to be reviewed. Their computation results of predicting the long-term performances of flat plate collectors used in typical space heating, cooling and hot water systems in Hong Kong are then compared. In the present analysis, only daily insolation data have been used and all the hourly values are derived from the daily insolation data using the Liu and Jordan's correlations.[9]

THE ϕ-CURVE METHOD

The rate of useful energy collection by a flat plate solar collector, q_u, relating to the collector design parameters and meteorological conditions is given by the Hottel-Whillier[10, 11] equation as follows:

$$q_u = AF_R\ [I_T\,(\tau\alpha) - U_L\,(T_i - T_a)]^+, \tag{1}$$

where A = collector area,
F_R = collector heat removal efficiency factor,
$(\tau\alpha)$ = effective product of the transmittance of the transparent covers and the absorptance of the collector plate,
I_T = rate of total radiation incident on the collector surface per unit area,
U_L = collector overall heat loss coefficient,
T_i = inlet collector fluid temperature, and
T_a = ambient temperature.

The + superscript in equation (1) is used to indicate that only positive values of the bracketed quality are to be considered as useful energy collected. In other words, solar radiation must be above a critical level, I_c, in order for a flat plate collector to yield a useful energy gain. I_c can be found by setting equation (1) to zero:

$$I_c = U_L\,(T_i - T_a)/(\tau\alpha) \tag{2}$$

The rate of useful energy gain, q_u, in equation (1) can thus be written as:

$$q_u = AF_R\,(\tau\alpha)\,(I_T - I_c). \tag{3}$$

The long-term average energy collected, $\bar{q}_i$, for the i-th hour of the day, averaged over a period of N days is:

$$\bar{q}_i = AF_R\,(\tau\alpha)\,\frac{1}{N}\sum^{N}\,(I_T - I_c)^+ \tag{4a}$$

The plus subscript in equation (4a) is again used to indicate that only positive values of $(I_T - I_c)$ are considered. The total useful energy gained over a month Q_u, is simply given by:

$$Q_u = N\sum^{n}\bar{q}_i, \tag{5}$$

where n is the number of hours in a day.

In general, equation 4(a) can be written as:—

$$\bar{q}_i = AF_R\,(\tau\alpha)\,\bar{I}_T\,\phi, \tag{4b}$$

where $\bar{I}_T$ is the long-term average hourly radiation incident upon the collector surface, and ϕ is defined as the fraction of the long-term average hourly radiation, I_T, which is above the critical level, I_c.

$$\phi = \frac{1}{N}\sum^{N}\,(I_T - I_c)^+/\bar{I}_T \tag{6}$$

Hottel and Whillier suggested that ϕ be named "utilizability". It denotes the fraction of the incident radiation that can be collected or utilized by an ideal collector with $F_R = 1$, $\tau\alpha = 1$. Furthermore, the utilizability can never be equal to unity unless the heat loss from the collector can totally be prevented, and this is impossible to achieve in practice.

Whillier[4] has shown that ϕ is a unique function of $I_c/\bar{I}_T$. Using statistical analyses of solar radiation data for many different locations, Liu and Jordan[5] further showed that plots of ϕ versus $I_c/\bar{I}_T$ are a unique function of the clearness index $\bar{K}_T$ for any months or locations. The clearness index, $\bar{K}_T$ is the ratio of the monthly average daily total radiation on a horizontal surface to that of the monthly average daily extraterrestrial radiation. $I_c/\bar{I}_T$ is also known as the critical intensity ratio, x_c. Typical plots of the utilizability or ϕ curve versus the critical intensity ratio x_c at various values of $\bar{K}_T$ and for surfaces with different tilt angles with the horizontal, as characterized by the conversion factor for daily direct radiation $\bar{R}_D$, are well documented.[5, 6] Examples illustrating the use of the ϕ curves have also been presented in references.[5, 6]

THE $\bar{\phi}$ CHART METHOD

The total useful energy gain over a month or an extended period, Q_u, can be determined by summing the hourly contribution over the entire month. Alternatively, equation (5) can be written as:–

$$Q_u = AF_R\ (\overline{\tau\alpha}) \sum^{N} \sum^{n} (I_T - I_c)^+, \tag{7}$$

where $(\tau\alpha)$ is the monthly weighted average transmittance absorptance product for the collector; n the number of hours in a day and N the number of days in the month.

The total radiation incident on the tilted collector surface during the month can be specified as the product of the monthly average daily total solar radiation on the tilted surface, H_T, and the number of days in the month, N. In terms of hourly radiation,

$$\bar{H}_T N = \sum^{N} \sum^{n} I_T. \tag{8}$$

Q_u can also be rewritten in terms of $\bar{H}_T N$ as

$$Q_u = AF_R\ (\overline{\tau\alpha})\ \bar{\phi}\ \bar{H}_T N, \tag{9}$$

where ϕ is defined as

$$\bar{\phi} = \left[\sum^{N} \sum^{n} (I_T - I_c)^+\right] / \sum^{N} \sum^{n} I_T \tag{10}$$

$\bar{\phi}$ is known as the monthly average daily utilizatility. It denotes the fraction of total radiation during the month which is above a specified critical level. $\bar{\phi}$ has properties similar to ϕ and can be completely specified in terms of $\bar{K}_T$, and two other dimensionless quatities which are the collector orientation geometry factor, $\bar{R}/R_n$ and the critical radiation level, X_c.

The geometry factor, $\bar{R}/R_n$, accounts for the effects of collector orientation, and in general it varies with location and time of the year. $\bar{R}$ denotes the ratio of the monthly average radiation in a tilted surface to that on horizontal surface while R_n denotes the ratio of radiation on the tilted surface at noon to that on horizontal surface, for an average day of the month. Standard methods of calculating $\bar{R}$ and R_n are available in references.[7, 12]

The dimensionless critical radiation level, X_c, is given by:

$$X_c = I_c / r_{T,n} \; R_n \, \bar{H}, \tag{11}$$

where I_c is the critical level as defined in equation (2) and $r_{T,\,n}$ is the ratio of solar radiation at noon to the daily total radiation. The value of $r_{T,\,n}$ can be determined from graphs given in Liu and Jordan's paper.[6,9]

Typical plots of $\bar{\phi}$ versus X_c for various clearness indexes of $\bar{K}_T$ and geometry factors of $\bar{R}/R_n$ are given in Klein's paper.[7] It is to be noted that analytical representation of the $\bar{\phi}$ charts is also available,[7] and this permits the calculation of the $\bar{\phi}$ values to be completely performed on a computer or even a programmable hand calculator.

COMPARISON BETWEEN THE ϕ CURVE AND THE $\bar{\phi}$ CHART METHODS

A comparison between the results of the ϕ curve and $\bar{\phi}$ chart methods has been made by Klein[7] for a range of latitudes, critical levels and $\bar{K}_T$ values, and the discrepancies observed are reported to be small. However, it is desirable to compare the above two methods for some real space heating and cooling systems and hot water systems in a subtropical country in order to confirm that the discrepancies of the two computational results are in fact small. Two case studies for solar systems in Hong Kong are presented.

All Year Round Hot Water System

Hotels, restaurants, hospitals and public buildings in Hong Kong have a large demand for hot water for various purposes throughout the year. In typical solar hot water systems, the values of the flat plate collector's parameters are: $F_R = 0.90$, $U_L = 5.11\,\text{W/m}^2\text{K}$, $(\overline{\tau\alpha}) = 0.65$. $(\tau\alpha)$ is assumed to be 95% of the maximum value at normal incidence, $(\tau\alpha)_n$. The monthly average collector fluid inlet temperature is maintained constant at some minimum allowable value of 32°C. To maximize the solar energy collection throughout the year, the collector tilt angle, β, towards south is set equal to the latitude of Hong Kong which is 22.304°N. Other relevant meteorological parameters and solar radiation characteristics for typical months of the four seasons of Hong Kong are listed in Table 1.[13, 14]

In using the Liu and Jordan's ϕ curve method, the average values of the hourly radiation on the tilted surface, $\bar{I}_T$ and the corresponding critical levels x_c for the i-th hour of the day have to be first determined. R_D is then calculated based on the collector tilt orientation, $\beta = L$. Finally, ϕ is

Table 1
Meteorological parameters and solar radiation characteristics for Hong Kong

Month	$\bar{\delta}$ [degrees]	$\bar{\omega}_s$ [degrees]	$\bar{T}_a$ [°C]	$\bar{H}$ [MJ/m²d]	$\bar{K}_T$	$\bar{H}_d/\bar{H}$
Apr. (spring)	9.4	93.9	21.9	13.87	0.371	0.499
Jul. (summer)	21.5	99.3	28.5	18.84	0.479	0.389
Oct. (autumn)	– 9.6	86.0	24.9	15.33	0.491	0.378
Dec. (winter)	–23.3	79.8	17.5	12.51	0.519	0.355

Table 2
Calculation of ϕ values for typical collectors in a Hong Kong hot water system in the month of December

	Hours from noon					
	½	1½	2½	3½	4½	5½
$\bar{I}_T$ [MJ/m²h]	2.39	2.13	1.70	1.13	0.52	0.13
I_c [W/m²]	108	114	114	114	117	117
ϕ	0.87	0.82	0.76	0.67	0.32	0

obtained graphically from the generalized ϕ curves for a fixed monthly average value of $\bar{K}_T$. Table 2 shows the typical values of $\bar{I}_T$ and ϕ for the month of December and these values are assumed to be symmetrical about the solar noon.

Similarly, in using Klein's $\bar{\phi}$ chart method, the values of the geometry factor $\bar{R}/R_n$ and the critical radiation levels X_c have to be predetermined. Typical values of $\bar{R}/R_n$, X_c in July for instance have been evaluated as 0.98 and 0.043 respectively. The corresponding value of $\bar{\phi}$ is then obtained either analytically or graphically from the $\bar{\phi}$ charts for the average monthly value of $\bar{K}_T$.

The values of ϕ obtained by the Klein's ϕ chart method is then compared with the weighted average of the hourly values of ϕ, denoted by $\bar{\phi}'$ which is defined by:–

$$\bar{\phi}' = [\sum^{n} \phi \, \bar{I}_T] / \sum^{n} \bar{I}_T \tag{12}$$

Table 3 shows a comaprison of the calculation results. It will be observed that the values of the

Table 3
Comparison of the ϕ curve and $\bar{\phi}$ chart results for hot water heating systems in Hong Kong

Month	Liu & Jordan's ϕ-curve method	Klein's $\bar{\phi}$ chart method
	$\bar{\phi}'$ (eqn 13)	$\bar{\phi}$
Apr. (spring)	0.77	0.78
Jul. (summer)	0.93	0.94
Oct. (autumn)	0.88	0.88
Dec. (winter)	0.76	0.75

$\bar{\phi}$ calculation from Klein's $\bar{\phi}$ charts agree well with the value $\bar{\phi}'$ obtained from that of the Liu and Jordan's method to within ± 2% for all the typical months of the four seasons in Hong Kong.

Summer Cooling and Winter Heating System

In Hong Kong, there is a need for summer cooling from May to September and winter heating from November to February. In a typical solar cooling system, the collector fluid inlet temperature has to be as high as 78.5°C in order to drive an absorption chiller. On the other hand, in a winter heating system the collector fluid inlet temperature can be as low as 27.5°C. In this analysis, the same collector parameters as in the preceeding case study are to be used. But, the collector tilt angles are set at $\beta = L - 15°$ in the summer and $\beta = L + 15°$ in the winter, towards south to maximize the respective summer and winter solar energy collection.

For the present case study, the comparison of the results is shown in Table 4. It is to be observed that about 5.6% discrepancy is obtained in the case of cooling and only 1.2% discrepancy in the case of heating. There seems to be a trend of increasing discrepancy with decreasing utilizability. The discrepancies are mainly due to the difficulties in interpolating the values of ϕ from the Liu and Jordan's ϕ curves and the existence of some asymmetries in the Hong Kong hourly total radiation data around solar noon.

Table 4
Comparison of the ϕ curve and $\bar{\phi}$ chart results for space cooling and heating systems in Hong Kong

Month	Mode	Liu & Jordan's ϕ-curve method $\bar{\phi}'$ (eqn 13)	Klein's $\bar{\phi}$ chart method $\bar{\phi}$
Jul. (summer)	cooling	0.36	0.34
Dec. (winter)	heating	0.83	0.84

CONCLUSION

In this paper, both the Liu & Jordan's ϕ-curve and Klein's $\bar{\phi}$ charts methods for predicting the long-term performances of flat plate collectors in solar energy systems has been reviewed. Both methods are directly applicable for solar energy systems with a large storage capacity or a restricted range of collector fluid inlet temperature. Klein's $\bar{\phi}$ charts provide more or less the same information as the Liu and Jordan's ϕ curves, but with substantially less calculation effort.

Comparisons have been made of the two methods to predict the long-term performances of solar collectors used in the Hong Kong hot water systems, for summer cooling and winter heating systems. The agreement of the results obtained by the two different approaches is in general quite good. The maximum discrepancy for cooling systems is observed to be about 5.6%, while that of heating or hot water systems is less than 2%. There appears to be a trend of increasing discrepancy with decreasing utilizability.

NOMENCLATURE

A	=	collector area, m^2
F_R	=	collector overall heat removal efficiency factor
$\bar{H}$	=	monthly average daily total solar radiation on a horizontal surface, $MJ/m^2 d$
$\bar{H}_d$	=	monthly average daily diffuse radiation on a horizontal surface, $MJ/m^2 d$
$\bar{H}_T$	=	monthly average daily total radiation on the tilted collector surface, $MJ/m^2 d$
I_c	=	critical level defined by eqn (2), W
I_T	=	hourly (or rate of) total solar radiation incident on the collector surface, W/m^2 or $MJ/m^2 h$
$\bar{I}_T$	=	monthly average value of the hourly (or rate of) total solar radiation on the collector surface for a particular hour of the day, W/m^2 or $MJ/m^2 h$
$\bar{K}_T$	=	ratio of the monthly average total to the monthly average extraterrestrial radiation on a horizontal surface.
L	=	latitude, degrees
n	=	number of hours from sunrise to sunset in a day
N	=	number of days in a month or a specified period
q_u	=	rate of useful energy collected, W
$\bar{q}_i$	=	average hourly useful energy collection for a given hour of the day, MJ
Q_u	=	total useful energy gain, MJ
$r_{T,n}$	=	ratio of the radiation at noon to the daily total radiation, d/h
R_n	=	ratio of radiation on a tilted surface to that on a horizontal surface at noon
$\bar{R}_D$	=	ratio of the monthly average daily beam radiation on a tilted surface to that on a horizontal surface
$\bar{R}$	=	ratio of monthly average daily total radiation on a tilted surface to that on a horizontal surface.
T_a	=	ambient temperature, °C
$\bar{T}_a$	=	monthly average ambient temperature, °C
T_i	=	collector fluid inlet temperature, °C
U_L	=	collector overall energy loss coefficient, $W/m^2 K$
x_c	=	critical intensity ratio, $I_c/\bar{I}_T$
X_c	=	dimensionless critical level defined by eqn (11)

Greek Symbols

β	=	tilt angle of the collector surface from horizontal, degrees
$\bar{\delta}$	=	monthly average solar declination, degrees
$(\tau\alpha)$	=	effective product of the transmittance of the cover system and the absorptance of the collector plate

$(\tau\alpha)_n$	=	transmittance-absorptance product for radiation at normal incidence
$(\overline{\tau\alpha})$	=	monthly average transmittance-absorptance product
ϕ	=	hourly collector utilizability
$\overline{\phi}$	=	average daily collector utilizability defined by eqn (6)
$\overline{\phi}'$	=	average daily collector utilizability defined by eqn (12)
$\overline{\omega}_s$	=	monthly average sunset hour angle on a horizontal surface, degrees

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